Deriving Maxwell's Equations From An Inspiring Walk In The Hills
Robert Watson and Thomas L. Selby, Ph.D.

Go directly to the text of the paper.

Abstract

An argument in reasoning using only experience is proposed. The aim of this work is to prompt the reader to step outside conventional boundaries related to understanding our universe and analyze the world primarily through personal experience. Hindsight is used as an analytical tool to demonstrate the possibility of this occurrence and references are made to the area of physics and Maxwell's equations. This work stretches the boundaries of scientific reasoning, both in format and convention, to demonstrate to the reader the value of one's own experience towards a greater understanding of physical laws. Phenomena of light are used as a point of interest, but any naturally occurring observable would suffice. Simplicity is demonstrated as a fundamental part of the solution, and reasoning in one's own experience is reinforced. This is done, not in a manner that is meant to replace, or even challenge, current scientific methods. One of the goals of this work is to assist *the observer* in moving beyond what might be believed to be fact, but instead is a limiting factor in the progress toward understanding something far greater.

Modification des équations de Maxwell à la suite d'une marche inspirante dans les collines
Robert Watson and Thomas L. Selby, Ph.D.

Résumé

Un texte argumentaire lié au raisonnement basé uniquement sur l'expérience est proposé. L'objectif de ce travail est de forcer le lecteur à aller au-delà des frontières conventionnelles qui touchent à la compréhension de notre univers et à analyser le monde principalement à l'aide d'expérience personnelle. Le recours à la distanciation, outil analytique utilisé pour démontrer la possibilité de cette réalité, et les références aux domaines de la physique et des équations de Maxwell font l'objet de cet article. Ce travail permet de repousser les limites du raisonnement scientifique, à la fois la forme et la convention, pour démontrer au lecteur la valeur de l'expérience individuelle par rapport à la compréhension globale des lois de la physique. Le phénomène de la lumière est l'un des points d'intérêt utilisé, mais tout phénomène naturel observable qui se produit pourrait être suffisant. On y présente la simplicité comme l'une des parties fondamentales de la solution et on accentue l'importance du raisonnement fondé sur l'expérience individuelle. Cette argumentation n'a pas pour objectif de remplacer ou même de s'opposer aux méthodes scientifiques actuelles. L'un des objectifs de ce travail est d'aider *l'observateur* à aller au-delà de ce qui peut être perçu comme un fait, mais qui en fait est un facteur qui freine l'acquisition d'une compréhension encore plus évoluée du monde qui nous entoure.

Derivando las Ecuaciones de Maxwell de una Inspiradora Caminata por las Colinas
Robert Watson and Thomas L. Selby, Ph.D.

Resumen

Se propone una argumentación de razonamiento utilizando solo la experiencia. El objetivo de este trabajo es incitar al lector a salirse de su zona convencional del entendimiento con relación a la comprensión del universo y analizar el mundo primordialmente a través de la experiencia personal. La comprensión retrospectiva se utiliza como una herramienta analítica para demostrar la posibilidad de esta ocurrencia y se utilizan referencias al área de la física y las ecuaciones de Maxwell. Este trabajo amplia las limitaciones del razonamiento científico, tanto en formato como en convencionalismos, con el fin de demostrar al lector el valor de la experiencia propia para el entendimiento de las leyes físicas. Se utilizan fenómenos de la luz como un punto de interés, sin embargo cualquier fenómeno natural observable sería suficiente. Se muestra la simplicidad como una parte fundamental de la solución y el razonamiento de la experiencia propia lo refuerza. Esto se realiza, no para reemplazar o siquiera desafiar los métodos científicos actuales. Uno de los objetivos de este trabajo es ayudar *al observador* a ir más allá de lo que se cree que son Hechos, pero en realidad, se trata de un factor limitante en el progreso hacia la comprensión de algo mucho mas grande.

Derivando as Equações de Maxwell a Partir de uma Caminhada Inspiradora pelas Colinas

Robert Watson and Thomas L. Selby, Ph.D.

Resumo

É proposto aqui um argumento sobre o raciocínio de se utilizar somente experiências. O objetivo deste trabalho é alertar o leitor para ir além dos limites convencionais relacionados ao entendimento de nosso universo e analisar o mundo, primeiramente através da experiência pessoal. A percepção é usada como uma ferramenta analítica para demonstrar a possibilidade desta ocorrência e são feitas referências à área da física e das equações de Maxwell. Este trabalho amplia os limites do raciocínio científico, tanto no formato quanto na convenção, para demonstrar ao leitor o valor da própria experiência de uma pessoa em relação a um entendimento maior das leis da física. Os fenômenos da luz são usados como um ponto de interesse, mas qualquer variável que ocorre naturalmente seria suficiente. A simplicidade é demonstrada como uma parte fundamental da solução, e é reforçado o raciocínio sobre a própria experiência de uma pessoa. Isto é feito não de uma maneira que tem como objetivo substituir, ou mesmo desafiar, os métodos científicos atuais. Um dos objetivos deste trabalho é ajudar o *observador* a se mover além do que poderia ser acreditado como verdadeiro mas, ao invés, ser como um fator limitante no progresso em direção ao entendimento de algo muito maior.

Die Ableitung von Maxwells Gleichungen aus einem anregenden Spaziergang in den Bergen

Robert Watson and Thomas L. Selby, Ph.D.

Zusammenfassung

Wir stellen eine vernünftige Diskussion die nur durch Erfahrung basiert ist, vor. Das Ziel dieser Arbeit ist den Leser dazu zu bewegen, außerhalb der konventionellen Grenze unseres Verständnisses zu treten und die Welt primär aufgrund von persönlicher Erfahrung zu analysieren. Der Blick zurück wird als analytisches Werkzeug benutzt um die Möglichkeit der Realität des Vorfalls zu demonstrieren, und es wird Bezug genommen auf das Gebiet der Physik und Maxwells Gleichungen. Diese Arbeit streckt die Grenzen der wissenschaftlichen Überlegung aus, sowohl in der Größenordnung als auch der Konvention, um den Leser den Wert der eigenen Erfahrung hin zu einem besseren Verständnis der physikalischen Gesetze zu demonstrieren. Als interessantes Detail werden Lichtphänomene benutzt aber jedes natürlich vorkommende Phänomen würde ausreichen. Als fundamentaler Teil der Lösung wird Einfachheit demonstriert, und was die eigene Erfahrung angeht, sollte bei dieser Vernunftsüberlegungen verstärkt werden. Dies soll nicht in einer Weise stattfinden dass gegenwärtige wissenschaftliche Methoden ersetzt oder auch in Frage gestellt werden. Eines der Ziele dieser Arbeit ist *dem Beobachter* jenseits zu Unterstuetzen und zu helfen mit dem was als Tatsache geglaubt werden

koennte, was aber ein limitirener Faktor ist auf dem Weg hin zum Verstaendnis von was viel Groesser ist.

Introduction

Hindsight is a wonderful thing! With this very thought in mind, let's reflect on our present knowledge and peer into the future of what might be known.

Natural Philosophy

Here will be presented an argument in the spirit of natural philosophy that is not entirely empirical in the traditional sense. The aim is to guide the reader to question what empiricism actually is and how these ideas are used today to form what is believed to be "true" in scientific thought. What is suggested here is provocative heresy, but is to be read with an open mind, not because it is to be deemed "right" or "wrong," but simply because it raises issues and points of discussion which may be ignored by our modern philosophy of science. In this work, we are not confining ourselves to the philosophies of John Locke and David Hume (Locke 1894 and Hume 1748), or any scientific "method" based on hypotheses and rigorously reproducible experimentation. Instead we are looking into nature as our personal laboratory and relying solely upon our own judgment of our personal experience. In a sense we are following our own inner guide and interpreting the world in broader terms, which steps outside the traditional reductionist viewpoint. It is a light-hearted analysis, but it nevertheless raises unanswered, unasked, questions. By curious paths we shall derive Maxwell's laws defining the physical laws of electromagnetic force (Maxwell 1865), or at least be led to hypothesize them from an almost *a priori* argument just as the Aristotelians would have wanted to do. This is done to challenge the philosophy of the scientific method and its definition, its self-imposed limitations, its assumptions and presumptions—but not to replace it. We seek rather to enhance its depth of enquiry and step outside the box. Indeed, it is a call back to the simple truth of Aristotle's syllogisms (Smith 1989) as applied to empiricism, and how this method can be extended to the maximum in the case of fundamental physics.

Let us take a stroll in the hills, let us observe the flow of the eddy the stream, the river, the waves emitted by a stone thrown into water, their reflection and even interference. Let us sit on the heath at night and watch the moon and the stars, and planets revolve about us as we observe from the apparent motionlessness of our position, seemingly oblivious to the angular motion we are undergoing. Let us observe the changes in intensity of a camp fire as we add firewood and feel the corresponding changes in temperature and sense the correlation between light and heat. Let us observe the light emitted or reflected by any source, whether by day or night; the rainbow and its relationship to the Sun as we inspect a waterfall from different angles; the reflection of our image on the surface of a pool; or the refraction that distorts our depth judgment as we grasp for a fish in the stream. Let us observe a flash of lightning and the odd delay that the lightning and the thunder present us with and seek to understand why this delay occurs. Could we desire a better laboratory for the observation of nature than nature itself? Let us observe all this and see how far we might get with little but pure mathematics, recognition of the power of empiricism and a little pragmatism to guide us in the understanding of these phenomena. Phenomena eternally available, or so we must hope, for all humanity to observe, record, and wonder. Let us apply natural philosophy alone without sophisticated experimental equipment, with no more accuracy of measurement than one can construct with one's hands from forest wood and stone in a brief sojourn, and the few but powerful laws of nature bequeathed to us by Newton, which in any case we could prove to ourselves quite quickly with limited tools, assuming we knew the mathematics. But let us also add another ingredient, a sense of unity in that which we observe, that will manifest as a sense of mathematical elegance, and see if it can aid us in our quest for understanding. That is, we will seek out the simplest or most elegant description, the description that has the greatest explanatory power.

Whilst this approach is not necessarily a practical way forward for science, the time has come to reappraise our assumptions about empiricism, because empiricism, rightly, has become the dominant approach used to investigate physical reality. Above all it may be possible to learn something from the methods of the ancient Greeks, and apply a more personal philosophical approach to fundamental science. In order to do this we will take a fresh look at Newton's laws (Newton 1726), electromagnetism and other steps in the development of fundamental science. This will be done in a flash, or a seeming flash, relative to the historical developments that took place in these sciences over the years, which were long and tortuous, taking decades of work by large numbers of scientists. It will also be fast relative to any normal steps in the process of learning these systems or applying them. We will move over and through these momentous theories at break-neck speed as if they presented no hindrance at all, extracting only the philosophical insight we need for our argument. What we are doing will not therefore be rigorous in the usual sense, and it cannot be claimed to be "proven." However, it will be an *intuitive* grasping nevertheless, with its own internal logic and merit. We will in particular start with the Newtonian conception of physics.

The Nature of Light

Let us assume that we already know experimentally, or by careful observation of nature, all about inertial frames from a Newtonian conception, that is, elementary mechanics, and the laws that go with it (Borowitz 1968). We have two possibilities for inertial frames that make physical sense based on this, the original Newtonian one and the later one of special relativity (Einstein 1905). Both work with the data hypothesized, that is a Newtonian limit, and the evidence presented casually by our walk in the hills. But now let's say we want to fit the phenomenon of light into this framework. Since we can look at light reflected and refracted, it reasons that it is composed of some sort of wave made of component frequencies. If we have any doubts about this, the hills furnish us with ample opportunities to experiment with waves in water and the play of light on water so as to see the similarity between reflections and refractions in water and in light. In addition, we can see the beautiful and paradoxical display of the rainbow, that would require of us some analysis of pigmentation with local flora in order to unravel the complexity of our own senses. We should not consider too deeply ideas that light might be corpuscular, despite the fact that corpuscular versus wave models go back to the ancient Greeks, let us instead try to explain only what we can directly observe at this point, and that is waves. Let us imagine ourselves 300 years ago or more, with nothing but curiosity and nature to guide us, but with one important historical anachronism, one impossible gift from the future: every conceivable mathematical and methodological and philosophical tool, every future paradigm without prejudice, so that we may effortlessly select the most appropriate for our needs. But no technology! Instead, our gift is a rather unfair amount of hindsight.

Methods

What sort of wave is this phenomena we call light? What can be deduced from our own personal observations and limited technology?

Light as Phenomena

We know that some aspect of light can be more or less added, that is summed together. It may not be absolutely clear that this is in any sense linear, but linearity is a good first approximation, at least for our sought-for quantity that is hypothesized to underlie the phenomenon of light. There is an assigned zero intensity, and then there are gradations of relatively higher and lower intensities. There are no negative intensities because we have referenced, or calibrated, our zero intensity to a fixed scale from which we will analyze all phenomena. To a first approximation we have, underlying light, something that can be added together and multiplied by a real number. Secondly this underlying something must be invariant in inertial frames in some meaningful sense—it must therefore be a tensor—in the same three-dimensional space that we typically observe objects; otherwise all sorts of contradictions or at least complexities will arise. Inertial frames do not make sense without tensors (Lovelock 1989 and Bishop 1980) if we are to use generalizations of geometry. How arbitrary (and therefore complex) it would be for key physical geometrical objects not to be tensors, even in the absence of curvature. Notwithstanding the fact that Newtonian physics does not really pass this test, with our mathematical advantages, we must surely look at tensors anyway. This is the unfair advantage we bring with us to our trek: if we are to see the world geometrically, which is to say, naturally, we must look for natural—tensorial—descriptions. If necessary we might have to change Newton's laws to make it work!—and this is exactly what Einstein did.

A Geometrical Approach

This geometrical approach is of course a paradigm possible only since Riemann (Reimann 2004), and not actually integrated into science historically until Einstein. And indeed putting such an argument ahead of that which is understood to be empirical fits no feasible working paradigm. Historically, and empirically, it could never have happened like this. Nevertheless, Einstein made this change, although he based his theories on considerably more robust empirical data than a mere walk in the hills. Thus, the argument could have been constructed from geometrical considerations alone, almost *a priori*, without any of the solid evidence that was actually used historically. It could have been proposed from an argument such as the one presented here. And this is not something usually discussed by empiricists, perhaps because it smacks of *a priori* thinking. But is it?

The combination that is apparent in a rainbow (whether in the sky or a waterfall, as one might observe from a walk in the hills) is clear evidence that the underlying "something" directed from the Sun is constituted from many frequencies also added together in some way. We seem to be at least dealing with real number fields in this case. It would be difficult to argue at this stage that complex numbers or non-linearity need come into play, even if in some final analysis they do. Let us be both practical and far-seeing and seek the simplest, most elegant options whilst we still can. That our predecessors chose simple vectors instead of tensors reflects a partial attempt in this direction, but the tensors of differential geometry are simpler in some sense than the arbitrary space-plus-time vectors of Euclidean space-plus-time. To think otherwise is to place a physical notion of simplicity above a mathematical one. In this physical approach to the world, addition is more complicated than the complex fluid dynamics of drinking a glass of water. Just ask any child about this difficulty. But mathematically the opposite is true. If humanity were more mathematically-minded would it not be clear to the casual hill-walker that tensorial objects are in fact required?

Given four-dimensional space we must associate light (or the underlying phenomena which we are hypothesizing) with a tensor on a four-dimensional space. Maybe a vector, maybe a scalar, maybe a metric tensor, maybe some convoluted gauge theory—or maybe a differential form (whether in a Euclidean or Minkowskian/Lorentzian space). So we can at least fix our search space to this mathematical region if we wish to develop a first approximation theory of light from our walk in the hills. We need not bother with curvature of space-time, unless we consider refraction a function of it. But this does not tally with our *experience* of mechanics. In any case the lightning's flash is far faster than thunder; curvature does not seem particularly relevant to light.

From hindsight we are cheering the differential form team, a specific subset of tensorial objects, which in any case encompasses the vector and scalar options to some extent. But our hindsight must be partially put aside for this derivation; we must still find the logical way through to the modern position if such exists. Any tensorial object at this point is as good as the other, although perhaps we might favor simple over complex explanations: discounting tensorial objects of greater dimensionality than four, discounting complex numbers, complex vector bundles, complicated gauge theories and so on. If it turns out we could obtain two equally valid explanations, we would start with the simple one as a preference. So pragmatically we can reduce our search to real tensors of no more than rank four, looking further only if no satisfactory model arises, and discounting the more complicated options if simpler ones work.

Mathematical Models and History

As perhaps calculus was the stumbling-block to the Greeks who failed to derive Newtonian mechanics from their observations, one wonders if the difficult step for scientists in the eighteenth and nineteenth centuries with respect to Maxwell's laws might not have been a lack of differential geometry, in particular differential forms (Kühnel 2002 and Lovelock 1989). This includes, of course, their attachment to a practical sense of space and time. They remained wedded to a practical vector description of electricity and magnetism (or at least a non-differential form description) even until after Einstein. Yet any sufficiently astute mathematician trained in twentieth-century mathematics, regardless of experimental sophistication, could reach the point thus described from a mere walk in the hills. And one wonders quite how much of this need in fact be hindsight after all. Mathematical tools are powerful things. Perhaps we could imagine a Cartan living amongst the Greeks doing just this and discussing his mathematical tools with Pythagoras? But doing this we are demeaning the importance of the vast historical development of pure mathematics upon which, in truth, each generation builds only its small layer. Were it so easy, the Greeks would have at least had Newton's laws. But they didn't. Nevertheless, such would have been easily within their technological and experimental capabilities, if any real technology is needed at all—it wasn't the technology that was limiting them in this regard, and they weren't far off in terms of mathematical sophistication either. Perhaps even more significant here is the philosophical position of the modern with respect to the Greek, the faith we have in our experience that mathematics can, or could, describe all that there is in physical form, and the confidence and accuracy and plain usefulness that comes from the empirical scientific method which although available to the Greeks was not so well-developed at that time.

Phenomena, Models, and Simplicity in Reasoning

Where possible, the Greeks preferred the *a priori* reasoning similar to our walk in the hills (that which we are entertaining here). Such a method cannot be advocated as efficient or even humanly possible as an approach to advancing science. However, there are many things we have mistakenly thought not humanly possible only to be later entertained and educated by our own ignorance—as many spectacles of human performance do almost routinely. But let us out of curiosity see just how powerful their methods could have been, in theory at least, and integrate something from it in our own quest. So, we must ask, which tensor best describes light—or that phenomenon which we are hypothesizing lies behind light? And in which inertial system do we observe: Euclidean or Lorentzian? Our options still appear wide open.

We can close down these options with a bit of astuteness based on principles already known, and by appealing to mathematical simplicity. We will want conservation laws to be present in the physical system governing light, at the very least conservation of energy—this follows from our Newtonian conception where conservation of energy, momentum and angular momentum loom large. There are many ways to do this with tensors: the metric tensor has zero covariant derivative, the Einstein tensor is source-free, differential forms have a zero double exterior derivative, that is the derivative of closed forms is zero, and we could add in by hand, artificially, a conservation law defined for almost any tensor. And given various further constraints there are no doubt many other ways—such as the Bel-Robinson tensor, which is conserved on 4D Ricci flat Lorentz manifolds, and other super-energy tensors. Of all of these, however, the two simplest and above all self-contained methods are to consider either the always-conserved Einstein tensor and the conserved closed differential forms, in that their exterior derivatives are zero. And it is the simpler possibilities that should manifest in nature with the simplest consequences, and thus the most readily observed. All the other obvious methods require additional constraints, they lack naturalness, that is, simplicity and explanatory power, and can thus be discarded as not appropriate to a first attempt to explain light. The first implication of differential geometry being, as Riemann understood long before Einstein, that curvatures of metrics could be open to consideration as potentially describing reality, such as forces. And he did this on mathematical elegance grounds alone. Thus, even though the Einstein tensor is used to describe gravity (Misner 1973), we cannot assume such a posteriori knowledge, herein. For all we know such a natural tensor may describe light waves rather than gravity waves—and if we appeal to mathematical elegance from the knowledge acquired only through our walk in the hills, such a natural way to produce waves must also be initially allowed in the mix.

In any case we have reduced our search space of the underlying field to a finite list of the "simplest" possibilities likely to arise on our casual stroll: Einstein's tensor (here given labels) in either (Ei) Euclidean or (Eii) Lorentzian weakly-curved space or space-times; and 0, 1, 2, 3 or 4-forms, once again in either Euclidean or Lorentzian flat space or space-time (labelled F0i, F1i, F2i, F3i, F4i, F0ii, F1ii, F2ii, F3ii, F4ii, respectively). And nothing else—if we failed to find a suitable contender for our description of light using these simplest of options, then perhaps the next step would be an order of magnitude more complex, essentially intractable search space, at least for our walk in the hills, and indeed this is the case for modern physics of gauge theories, the search space being so much larger. In addition we ought to consider sound waves (Sei/Seii) and transversal displacement (Tei/Teii) waves in some sort of "ether", assuming that the conservation laws are taken care of essentially by a mechanical process not incompatible with Newton's laws, as early investigators assumed. Thus we have sixteen reasonable options at this point. We have at least narrowed the search space to something manageable.

Results and Analysis

Of course in historical terms we have just waved away most of the history of science in this analysis. For many decades scientists assumed a corpuscular model of light, let alone reducing the search space to tensors, let alone again to a few choice ones. With hindsight the path was *always* there, potentially to be done in a single wave of the hand, a single bat of the eyelid. Humanly possible? No! Only with the grossest hindsight is this possible for humans—but possible *in principle* it remains.

A Further Reduction of Options

In reducing these options we will first see if there are any obvious reasons to discount any, in the sense that some of the options may be contrary to observation. Take (Ei) and (Eii) as cases in point. We discount them because they require that light or some aspects of light distort the paths of other objects in motion, as gravity waves would. It might of course simply be that the effect is too weak to be readily noticed, as they are with gravity waves, but this does not remove the problem: we have another source of a force that appears to move the paths of all objects—the Earth, and this seems to emit very little light relative to a typical light source, such as say, a camp fire. It must surely be "stuff" or "matter" that is the source of curvature under these two hypotheses, yet there is no *apparent* connection between the Earth, or matter, and light. By the principle, or "sense," of unity, which we have hypothesized, this whole consideration leads to more questions than answers, both (Ei) and (Eii) can thus be readily discarded, at least according to our methods. But is this sense of unity justified? Nowhere is a sense of unity mentioned in any treatise on the scientific method! We must make observations based on carefully planned and executed experiments and deduce our laws from those results, all the while taking current scientific theories relevant to that field of investigation into consideration. This is the nature of scientific investigation. Surely our reasoning here is but exploiting hindsight and knowledge of general relativity? And of course it is, but where does the fault lie—in our lack of reasoning or our lack of empiricism? Einstein deduced general relativity from little more in reality than the Michelson-Morley experiment, no great body of new data at all, and a healthy dose of mathematical elegance. But it could have been hypothesized just one logical step before even that experiment had been undertaken in the mid 1800s. And the case here is given to suggest that a tensorial view, coupled with the sense of unity that that embraces, would have led to it even earlier, such that it could have been apparent even to the Greeks or, merely from our walk in the hills. This whole question is left as unanswered and maybe unanswerable, the objective here is to taunt the reader for his sureness of the scientific method, and maybe lay bear its assumptions. Because hidden deep within it are indeed assumptions, assumptions about what constitutes evidence and experiment and deduction, and they are assumptions without proof. They may not be fully correct even though they may work well.

Let us take (Sei/ii) and (Tei/ii) similarly. Either way we must have a parallel system of moving, oscillating components (as indeed do exist in the form of charges), that are to exist as it were in parallel with our mechanical system of masses and heat/sound vibration. Despite the fact that this is a correct view of what is actually happening within atoms and molecules, as light passes through a medium, we will discount it as not satisfying our sense of unity! The criteria of a sense of unity has surely led us astray. However, a finer analysis shows that this is not true. For we have discounted options (Sei/ii) and (Tei/ii) only relative to the other options with which we hope to display a higher degree of unity. And indeed through Maxwell's laws this will indeed take place. Or to put it another way, equally, we can accept (Sei/ii) or (Tei/ii) if and when we can find the unifying factor, which will be the electromagnetic field, and thus the argument is subsumed by the remaining options. And the remaining options will prove to be a more powerful starting point—and eventually lead to a transversal model (in some specific technical sense) of electromagnetic waves. It does not seem here that we are using hindsight, because the argument has specifically eliminated that which is true—despite the fact that it has not eliminated (yet?) that which is more true. The astute reader will have noticed that by this curious argument we have already reduced our search space to differential forms; a powerful mathematical tool not available in any state of refinement in the era of Maxwell, let alone our Newtonian hill-walker. Such a reduction to mathematical elegance is itself promising.

But of course an argument such as this cannot but be tainted with undue hindsight or even artificiality, it would need to be proven in the lab. But, let us not censor an argument on that account, on account of unavoidable imperfection, we may put forward a hypothesis and test it later. Above all let us not deny this argument on the basis that nobody would ever have done it like this, that it is historically absurd—the sheer possibility of such farsighted insight with such little data is all that is being discussed here, not its likelihood or feasibility, which is as already mentioned not likely humanly possible. As historical actuality it is quite impossible. Here however, all that is requested is an open mind simply to follow through the argument and see the point, and then dismiss it as useless or interesting as the case may be. Of course, any conclusions about the nature of reality from this method would need firm verification, so perhaps in any case all we are doing is identifying an approach that could have led to a good hypothesis, rather than a theory—and that may or may not be so. Given such provisos let us continue our walk in the hills after some time to stop, rest, and think.

Empiricism, Reductionism and Unity

Let us for a moment analyze the foundations of empiricism more directly, not in a negative sense, but more out of curiosity. The idea that we can observe something and draw conclusions from the observed data makes total sense, and is extremely powerful. Indeed instrumentation used to analyze data has advanced and become more powerful, but is still bound by our own understanding of both the design of the instrument and our ability to analyze the data the instrument is producing. While telescopes, lasers, and computers have extended our understanding, all roads to analysis, and hence empiricism, always come back to the observer. Further experimentation and hypothesis testing is then possible, and fundamental to science, enabling a feedback that has more or less led to our understanding of the world today. But deep in the foundations of this simple, common sense, and correct, practical principle, lie a few assumptions that should be teased out for inspection: how exactly do we draw conclusions from data we obtain, regardless of the source? Additionally, is that different from how we should or could draw conclusions from data from our own experience? In other words, what is the basis upon which we do and should discern patterns in data and, as it were, join the dots? The answer of course is as best one can with one's senses and intellect, always seeking simplicity and clarity. But herein lies the problem: our ability to find patterns is limited by our viewpoint (humanity), even our personal talents, and what is simple for us may not be simple "objectively," that is, to a more insightful other. Our concept of what a pattern is, a mathematical pattern in this case (which is bound up with this concept of simplicity) is defined to some extent by this subjectivity. We may, by utilizing the process of empiricism, be routinely missing patterns that some other intelligence would deem obvious, or merely tricky. And indeed that is what is happening when we fail to apply advanced but elegant mathematical tools, or even more broad sweeping philosophical tools, that do not yet exist or are not yet familiar to us, despite their potential. By way of analogy, every school child knows how difficult algebra can be, yet the construction of algebra is elementary and elegant as a mathematical concept. It is complex from one perspective, but simple from the other. It is mathematically powerful and elegant, but it is considered difficult and unnatural. How many more systems in our universe have this property? How much more the mathematical basis of relativity, familiar in detail to no more than 1 in 1000? Thus, at the very root of empiricism lies the concept of mathematical elegance, something difficult to define, and indeed elusive by its very nature; the ultimate truth of which must always lie beyond our grasp. And it is argued here, though this cannot be proven, that this is a part of the concept of a "sense of unity" as applied to the physical world to which the arguments here defer. Or to put it another way, seeking any pattern in nature is an implicit search for unity. The concept of unity being either (i) alien to modern empiricism as conceptually too *a priori*, or (ii) a part of empiricism only via our humanity, that is, as a human endeavor rather than an "objective" one. Either way, empiricism is not quite the same product as that which it is sold to be: pure objective rationalism. That does not exist, unless one is able to set aside any and all viewpoints prior to investigation.

We have here a fundamental criticism of a philosophy often used in science, which, while not strictly a part of empiricism often dominates it: the mindset of reductionism. What is proposed here is nothing less than a philosophical reason within empiricism why science should leave reductionism behind as a useful but now potentially misleading doctrine, or as a tool to be used, but not to be assumed in all cases. That reductionism is limited and may miss key interpretations that observed data in fact suggests. That reductionism by itself is a false assumption based on an over simplification of empiricism.

Discussion

What then of experimentation when reductionism is questioned? How does one investigate the fundamental laws of nature, and learn about the Universe around us, without devising carefully executed experiments that are isolated from any chance contamination that might bias the results (that is, without separating the elements reductively)? Additionally, is it possible to consider the principle of Unity while carrying out experiments in isolation? Are these not contradictions that arise once reductionism is challenged?

The idea of Unity is, by definition, not reducible to anything other than itself. At the same time, it includes all components. However, we, the observers of this elegant universe, have throughout history drawn boxes around natural phenomena and named the components inside for our own benefit, somewhat artificially. We create definitions that need to be expanded upon later, or need further mathematical treatment to be considered more accurate than the same measurements of yesteryear. We have measured the speed of light to great accuracy, yet in areas that are new to us, such as dark matter and dark energy, our previous conceptions of matter are once again defied. We devise the limitations and boundaries of our world, and then investigate further to remove them. An open mind and the search itself is the resolution of these contradictions. There can be no rule to say how one must search however, or how best to have an open mind. For if one knew, it would not be a search, and the mind would be already closed.

So, in this spirit let's continue our walk, we have tarried long on our hill top, and it is getting late and darkening as we reach the dusk of our day's journey. Let us keep our minds open if we dare, and our ideas free from chains if we are strong enough and of keen wit. From the grand vistas of the hills, we plunge now into the forest of details, where differential forms lurk in the shadows of the mathematical wilderness...

A Closer Look at Details: Differential Forms

We have in fact been led, albeit via long and varied paths, to surmise the importance of differential forms from everyday observations. And we may go further still. By noting the direction of lightning as having three dimensions of extension, we are looking for a 3 vector to express the force. Herein lies a problem: how do we get a 3-vector out of differential forms? In fact we have the same problem for gravity, but in this case we cannot discount curvature and we would be led by parallel arguments to general relativity, and the gravitational field as a 3-vector as a limit via the Einstein tensor of Lorentzian spacetime. There are two ways out (i) the electric force isn't a 3-vector at all but really a 4-vector and (ii) that the 3-vector isn't so much a 3-vector but three components of a 2-form, distributed as is the case in fact in the Faraday or Maxwell tensor, where three of the available components are taken by the electric field, and three by the “magnetic” field. We might ask, why go to this latter complexity when we could just have the force as a 3-vector arbitrarily placed or appended to space as was done by Maxwell? And the answer is simply that we are looking for a more fundamental explanation and the unity that comes from looking at, and for, genuinely natural mathematical objects, that is, genuine simplicity, not human simplicity. Thus: a 4-vector, yes, but a 3-vector, no. Or, let us say that a 3-vector makes sense with respect to Newton's laws, so that we in fact have some sort of displacement going on (as indeed with the charged ion in lightning), and we are led via a previous argument (the one regarding transverse waves) back to a search for the underlying field, in terms of the "simplest" possible mathematical objects, regardless. We would have to be looking at 4 vectors or differential forms, and the only way to make that work will turn out to be the Lorentzian signature of special relativity.

So, we now have two options (or four including both the Euclidean or Lorentzian variants). We can discount the 0-form (a simple real numbered function) as insufficient to code the three dimensionality of our sought-for force, although it could be the potential of our 1-form force. And we can discount 4-forms for the same reasons. The application of a "magnetic" field in a 2-form is possible without any loss of explanatory power, as a transverse wave uses up only half its available dimensions for oscillation (it uses only one of the two available axes). But with greater numbers of components this possibility reduces, or at least becomes far more complicated. We should first look for an explanation in terms of the electric field being described by a 1-form (as this offers the simplest model). Failing that, in terms of a 2-form (as indeed is the case). Discounting 3-forms only in that they are subsumed into 1-forms as their hodge dual.

And perhaps the point of this tall tale at this point in our journey is that the concept of genuine simplicity can be humanly complex, for although such perception as presented here with hindsight is quite possible, as said previously, no human being would ever likely have done things like this from the outset. Yet the argument continues.

At this point we can start to think about the Euclidean space plus time model (somehow represented by a 4D manifold), and the only viable and straightforward manifold alternative in 4 dimensions, that of a 3+1 Minkowskian/Lorentzian space-time as in special relativity. Now, given that we are looking at forms (a 1-form or a 2-form) to describe relativity, we must ask, if the case is Euclidean: what has happened to rotations from space to time and how do they represent changes of coordinates? We have, perhaps without knowing, simplified our reasoning into a trap by assuming differential geometrical objects, whereby the Euclidean model becomes untenable, and special relativity must result. The rotation from space to time has been brushed under the carpet, as an impossible maneuver. From a differential geometry perspective, Euclidean space plus time was a "complex" mathematical structure from the outset, regardless of how our brains are wired to understand it. That is, space and time as understood by Newton and Euclid are not single geometrical objects, but some more complicated convolution. Mathematically the unification of space and time in relativity, in a Lorentz manifold, makes a priori sense, and is far simpler in a deeper sense. The Greeks could have stumbled upon it with a bit more mathematical sophistication by almost a priori reasoning, but history left it to a more technological society rooted in the other side of the same coin: empiricism. How much better to combine both, as perhaps Einstein and others did. Could this use of such a syllogism, of synthesizing two sides of the same coin, two seeming polar opposites that are in fact one, be the key to breaking into new paradigms?

Maxwell's Equations as a Result

We are now left with strictly two options: a 1-form or a 2-form description of electric force within special relativity. The first would correspond to a scalar potential, the second to the vector potential of electromagnetism. We need to generate a wave, for which the simplest way for the 2-form is Maxwell's laws: in terms of the 2-form F, it is simply that the exterior derivative of the hodge star of F is 0; i.e. Maxwell's laws without source, which we know suffices, and that ample radiative solutions exist for our purposes. We notice that the simplicity comes from the fact that both F and *F are 2-forms, an occurrence that only happens for 2-forms in 4 dimensions. Finally applying the same hope to a 1-form force field, we observe no such simply apparent wave solutions. That possibilities exist is true without a doubt, but they are somewhat forced and require more "complexity" than the simpler Maxwell's equations. It is as if with Maxwell's equations the universe has chosen the only possibility available to it in terms of "simplicity," and mathematical elegance. At least given the data provided by a walk in hills. If one sees a few dots on a graph approximating a curve, one is empirically justified in lining them up, but the same thing can be said for far more mathematical objects than our limited brains are programmed to naturally perceive. Joining the dots of our observations of our walk in the hills, we might declare: "differential forms, tensors and a Lorentzian space-time metric." The fact that reality might comply with or be subject to such reasoning, or even allow us to construct such reasoning with hindsight, is peculiar. Yet this peculiarity comes from our own difficulty in perception, our own labeling and classifying, our reductionist ways. It is an artifact of our own thoughts. It is like being frightened by our own moonlit shadow as we exit the forest and start on the path back home.

What Have We Gained?

We have, despite much success with Maxwell's laws, not yet been led to hypothesize the Lorentz force law or the particle nature of the electron, or the non-existence of monopoles, or many other things fundamental to electromagnetism. The success presented here in devising an argument that appears to naturally lead to Maxwell's laws seems not to go any further. But Maxwell's laws (albeit source-free in this case) are far enough for the purposes. Perhaps it would be better to say that even if it did go further (as it may well, after all: cannot one look at the embers of a fire and consider their thermodynamics starting from Newtonian mechanics alone? Might not this lead to the quantum of action?), we would need in any case to test our deductions, to finally obtain empirical certainty from what would otherwise remain mere supposition based on astute observations. That is, what in any case remains is the need to do some tangible experiments and prove what we have surmised from observation, something at which modern science already excels. But yet one still wonders how far astuteness alone can go in forming hypotheses, and what this tells us about the real nature of empiricism. How the two sides of syllogism can be better harnessed. Seeing a simple pattern in data from an observation is not in fact a matter that is necessarily simple at all, for mathematical simplicity may humanly appear complex, even impossibly complex, at least at first, and nobody can be truly sure that at any one moment we are not simply being blind to a deeper simplicity of reality, an alternative joining of the dots, that makes far more sense. Perhaps it is this that one is looking for in the development of new fundamental physical theory, however much it must be (and it *must* be) confirmed by further experimental tests? In short, the basis of the analysis step of empiricism is that very mathematical elegance that could otherwise be criticized as unempirical, the method the Greeks tried to use without sufficiently developing experimental work. But at the same time that does not prevent such reasoning from being a powerful guide, the complementary other side of the coin to supplement an experiment. And although one could criticize the Greeks for this one-sided approach, this would be unfair by any human measure, and would show our cultural one-sidedness too, rooted in practical technology, because in theory at least the Greeks might have developed further their analytical methods to compensate, as exemplified by the argument presented here. And at the same time modern science could learn from a more truly philosophical approach to fundamental science, that is, we could take on board the philosophy of the Greeks and discover new powerful theoretical and philosophical tools in the process, and it may be that it is exactly a move towards such a heightened and inspired approach that is needed to break the current dead-lock between quantum mechanics and general relativity that seems currently so intractable to experiment or even standard theoretical speculations.

Conclusion

Thus we return by evening from our walk in the hills having crossed much interesting, sometimes difficult, yet always beautiful terrain, and feeling greatly inspired. And considering the work we have undertaken, feeling strangely rested. Our path now takes us home via familiar and pretty lanes, the sight and smell of flowers and fields, the distant sight of houses and the surprisingly friendly and welcoming presence of man and toil. We cast our eyes up to the heavens, and contemplate how the stars seemingly so insignificant and equal are each a distinct and vast world unto themselves separated from each other as much as from us by inconceivable distances. We glance back briefly at the hills and the forest and wonder where we have just been. And then again the Milky Way carelessly sprinkled above, we pause to imagine what strange paths may take us there tomorrow. For one journey is but the logical extension of another, long journeys but extensions and continuations of shorter ones.

But not today, as we reach the threshold of familiar lands, and streets; here our brief walk in the hills reaches its end.

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